

# Dipole polarizabilities of charged pions

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We discuss main experimental works, where dipole polarizabilities of charged pions have been determined. Possible reasons for the differences between the experimental data are discussed. In particular, it is shown that the account of the  $\sigma$ -meson gives a significant correction to the value of the polarizability obtained in the latest experiment of the COMPASS collaboration.

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## I. INTRODUCTION

Pion polarizabilities are fundamental structure parameters characterizing the behavior of the pion in an external electromagnetic field. The dipole ( $\alpha_1$  and  $\beta_1$ ) and quadrupole ( $\alpha_2$  and  $\beta_2$ ) polarizabilities are defined [1, 2] through the expansion of the non-Born helicity amplitudes of the Compton scattering on the pion over  $t$  at the fixed  $s = \mu^2$ :

$$M_{++}(s = \mu^2, t) = \pi\mu \left[ 2(\alpha_1 - \beta_1) + \frac{t}{6}(\alpha_2 - \beta_2) \right] + \mathcal{O}(t^2), \quad (1)$$

$$M_{+-}(s = \mu^2, t) = \frac{\pi}{\mu} \left[ 2(\alpha_1 + \beta_1) + \frac{t}{6}(\alpha_2 + \beta_2) \right] + \mathcal{O}(t^2),$$

where  $s$  ( $t$ ) is the square of the total energy (momentum transfer) in the  $\gamma\pi$  center of mass (c.m.) system,  $\mu$  is the pion mass,  $\alpha_i$  and  $\beta_i$  are electric and magnetic polarizabilities correspondingly. In the following the dipole polarizabilities are given in units  $10^{-4} \text{ fm}^3$ .

The values of the pion polarizabilities are very sensitive to predictions of different theoretical models. Therefore, an accurate experimental determination of these parameters is very important for testing the validity of such models.

The most of experimental data obtained for the difference of the dipole polarizabilities of the charged pions are presented in Table I.

The polarizabilities were determined by analyzing the processes of the high energy pions scattering in the Coulomb field of heavy nuclei ( $\pi^- A \rightarrow \gamma\pi^- A'$ ) via the Primakoff effect, radiative pion photoproduction from proton ( $\gamma p \rightarrow \gamma\pi^+ n$ ), and two-photon production of pion pairs ( $\gamma\gamma \rightarrow \pi\pi$ ). As seen from Table I, the data vary from 4 up to 40 and are in conflict even for experiments performed with the same method. In this paper we will consider possible reasons for such disagreements.

## II. RADIATIVE PHOTOPRODUCTION OF THE $\pi^+$ -MESON FROM THE PROTON

An experiment on the radiative photoproduction  $\pi^+$ -meson from the proton ( $\gamma p \rightarrow \gamma\pi^+ n$ ) was carried out at the Mainz Microtron MAMI [3] in the kinematical region of  $540 \text{ MeV} < E_\gamma < 820 \text{ MeV}$  and  $140^\circ \leq \theta_{\gamma\gamma}^{cm} \leq 180^\circ$ , where  $\theta_{\gamma\gamma}^{cm}$  is a polar angle in the c.m. system of the outgoing photon and pion.

The theoretical calculations of the cross section for the reaction  $\gamma p \rightarrow \gamma\pi^+ n$  show that the contribution of nucleon resonances is suppressed for photons scattered backward in the c.m. system of the reaction  $\gamma\pi \rightarrow \gamma\pi$ . Moreover, integration over  $\varphi$  and  $\theta_{\gamma\gamma}^{cm}$  essentially decreases the contribution of nucleon resonances from the crossed channels. In addition, the difference  $(\alpha_1 - \beta_1)_{\pi^\pm}$  gives the biggest contribution to the cross section for  $\theta_{\gamma\gamma}^{cm}$  in the same region of  $140^\circ - 180^\circ$ . Therefore, one considered the cross section of radiative pion photoproduction integrated over  $\varphi$  from  $0^\circ$  to  $360^\circ$  and over  $\theta_{\gamma\gamma}^{cm}$  from  $140^\circ$  to  $180^\circ$ ,

$$\int_0^{360^\circ} d\varphi \int_{-1}^{-0.766} d \cos \theta_{\gamma\gamma}^{cm} \frac{d\sigma_{\gamma p \rightarrow \gamma\pi^+ n}}{dt ds d\Omega_{\gamma\gamma}}. \quad (2)$$

The work was carried out at values of  $s$  up to  $15\mu^2$ . It has been shown in Ref. [20], that the contribution of the  $\sigma$ -meson is noticeable at such high values of  $s$ . The contributions of other mesonic resonances ( $\rho, a_1, b_1, a_2$ ) are negligible here.

The values of the pion polarizabilities have been obtained from a fit of the cross section calculated by two different theoretical models to the data. In the first model the contribution of all the pion and nucleon pole diagrams was taken into account. In the second model in addition to the nucleon and the pion pole diagrams (without the anomalous magnetic moments of nucleons) the contribution of the resonances  $\Delta(1232)$ ,  $P_{11}(1440)$ ,  $D_{13}(1520)$ , and  $S_{11}(1535)$  and the  $\sigma$ -meson were included.

It should be noted that the contribution of the sum of the pion polarizabilities is very small in the considered region of  $140^\circ \lesssim \theta_{\gamma\gamma}^{cm} \lesssim 180^\circ$ . The estimate shows that the contribution of  $(\alpha_1 + \beta_1)_{\pi^\pm} = 0.4$  to the value of  $(\alpha_1 - \beta_1)_{\pi^\pm}$  is less than 1%.

To increase the confidence that the model dependence of the result was under control, kinematic regions were

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TABLE I: Review of experimental data on  $(\alpha_1 - \beta_1)_{\pi^\pm}$ 

Experiments	$(\alpha_1 - \beta_1)_{\pi^\pm}$
$\gamma p \rightarrow \gamma \pi^+ n$ MAMI (2005) [3]	$11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}$
$\gamma p \rightarrow \gamma \pi^+ n$ Lebedev Phys. Inst. (1984) [4]	$40 \pm 20$
$\pi^- A \rightarrow \gamma \pi^- A'$ Serpukhov (1983) [5]	$13.6 \pm 2.8 \pm 2.4$
$\pi^- A \rightarrow \gamma \pi^- A'$ COMPASS (2007) [6]	$4.0 \pm 1.2 \pm 1.4$
D. Babusi <i>et al.</i> (1992) [7]	
PLUTO [8]	$38.2 \pm 9.6 \pm 11.4$
DM 1 [9]	$34.4 \pm 9.2$
MARK II [10]	$4.4 \pm 3.2$
J.F. Donoghue, B.R. Holstein (1993) [11]	5.4
MARK II [10]	
A.E. Kaloshin, V.V. Serebryakov (1994) [12]	$5.25 \pm 0.95$
MARK II [10]	
L.V. Fil'kov, V.L. Kashevarov (2006) [13]	$13^{+2.6}_{-1.9}$
$\gamma\gamma \rightarrow \pi^+\pi^-$ fit of data [10, 14–18] from threshold to 2.5 GeV	
R. Garcia-Martin, B. Moussallam (2010) [19], $\gamma\gamma \rightarrow \pi^+\pi^-$	4.7

considered where the difference between the models did not exceed 3% when  $(\alpha_1 - \beta_1)_{\pi^\pm}$  is constrained to zero. First, the kinematic region was considered where the contribution of the pion polarizability is negligible, i.e. the region  $1.5\mu^2 < s < 5\mu^2$ . Then the kinematic region was investigated where the polarizability contribution is biggest. This is the region  $5\mu^2 < s < 15\mu^2$  and  $-12\mu^2 < t < -2\mu^2$ . In the range  $t > -2\mu^2$  the polarizability contribution is small and this region was excluded.

Analysis of these data gave the following result

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{model}. \quad (3)$$

An independent analysis [21] of the experimental data was carried out by a constrained  $\chi^2$  fit. The result [21] agree very well with (3) giving it additional support.

The result [3] is consisted with earlier works investigating the  $\gamma p \rightarrow \gamma \pi^+ n$  [4] and  $\pi^- A \rightarrow \gamma \pi^- A'$  [5] reactions, and also with [13], where a global fit to all existing data for the  $\gamma\gamma \rightarrow \pi^+\pi^-$  reaction was done. On the other hand, the result [3] is in conflict with the prediction of ChPT [22, 23]. This discrepancy can be connected with a different account of the contribution of the  $\sigma$ -meson and vector mesons in the dispersion relations and ChPT calculations [24].

### III. SCATTERING OF PIONS IN THE COULOMB FIELD OF HEAVY NUCLEI

The first experimental data on the charged pion polarizability was obtained in the work [5]. They studied the scattering of high energy  $\pi^-$  mesons off the Coulomb field of heavy nuclei. A connection of the radiative scattering in the Coulomb field with the Compton scattering was first predicted in the work [25].

The cross section of the radiative pion scattering  $\pi A \rightarrow \pi \gamma A'$  via the Primakoff effect can be written as

$$\begin{aligned} & \frac{d\sigma_{\pi A}}{ds dQ^2 d\cos\theta_{\gamma}^{cm}} \\ &= \frac{Z^2 \alpha}{\pi(s - \mu^2)} F_{eff}^2(Q^2) \frac{Q^2 - Q_{min}^2}{Q^4} \frac{d\sigma_{\pi\gamma}}{d\cos\theta_{\gamma}^{cm}}, \end{aligned} \quad (4)$$

where  $F_{eff} \approx 1$  is the electromagnetic form-factor of nucleus,  $\alpha$  is the fine-structure constant,  $Z$  is the charge number of the nucleus, and  $Q^2$  is the negative 4-momenta transfer squared,  $Q^2 = -(p_A - p'_A)^2$ .  $Q_{min}^2$  is the minimum value of  $Q^2$  which is given by the formula

$$Q_{min}^2 = \frac{(s - \mu^2)^2}{4E_{beam}^2}, \quad (5)$$

where  $s$  is the square of the total energy of the process  $\gamma + \pi^\pm \rightarrow \gamma + \pi^\pm$ ,  $E_{beam}$  is the pion beam energy.

This cross section has a peak at  $Q^2 = 2Q_{min}^2$  with a width equal to  $\simeq 6.8Q_{min}^2$ .

The experiment [5] was carried at a beam energy equal to 40 GeV. In this case if the energy of the incident photon in the incident pion rest frame  $\omega_1 = 600$  MeV, then  $Q_{min}^2$  is equal to  $4.4 \times 10^{-6} (\text{GeV}/c)^2$ . It was shown that the Coulomb amplitude dominates in this case for  $Q^2 \leq 2 \times 10^{-4} (\text{GeV}/c)^2$ . The experiment [5] was carried out at  $Q_{cut}^2 < 6 \times 10^{-4} (\text{GeV}/c)^2$ . Events in the region of  $Q^2$  of  $(2 - 8) \times 10^{-3} (\text{GeV}/c)^2$  were used for estimation of the strong interaction background. This background was assumed to behave either as  $\sim Q^2$  in the region  $Q^2 \leq 6 \times 10^{-4} (\text{GeV}/c)^2$  or as a constant. The polarizability was determined from the ratio (assuming  $(\alpha + \beta)_{\pi^\pm} = 0$ )

$$R_\pi = \left( \frac{d\sigma_{\gamma\pi}}{d\Omega} \right) / \left( \frac{d\sigma_{\gamma\pi}^0}{d\Omega} \right) = 1 - \frac{3}{2} \frac{\mu^3}{\alpha} \frac{x_\gamma^2}{1 - x_\gamma} \alpha_\pi, \quad (6)$$

where  $\frac{d\sigma_{\gamma\pi}}{d\Omega}$  refers to the measured cross section and  $\frac{d\sigma_{\gamma\pi}^0}{d\Omega}$  to simulated cross section expected for  $\alpha_\pi = 0$ ,  $x_\gamma = E_\gamma/E_{beam}$  in the laboratory system of the process  $\pi A \rightarrow \pi\gamma A'$ . As a result they have obtained

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 13.6 \pm 2.8 \pm 2.4. \quad (7)$$

The new result of the COMPASS collaboration [6] for the charged pion electric polarizability  $\alpha_\pi = 2.0 \pm 0.6_{stat.} \pm 0.7_{syst.}$  has been found also by studying the  $\pi^-$ -meson scattering off the Coulomb field of heavy nuclei. The result was obtained assuming that  $\alpha_1 = -\beta_1$ . This value is at variance with the result obtained in a very similar experiment in Serpukhov [5], but also with [3].

The COMPASS experiment [6] was performed with  $E_{beam} = 190$  GeV. For such values of  $E_{beam}$  the quantity of  $Q_{min}^2$  (COMPASS) must be smaller by 22.5 times than  $Q_{min}^2$  (Serpukhov). In this experiment [6] the authors considered  $Q_{cut}^2 \lesssim 0.0015$  (GeV/c) $^2$ .

As shown in [26] the basic ratio  $R_\pi$  is applicable for the Coulomb peak only. In Ref. [27] it is elaborated that the Coulomb amplitude interference with the coherent nuclear amplitude is important for  $0.0005 \leq Q \leq 0.0015$  (GeV/c) $^2$ . This means that the Serpukhov analysis could safely apply the ratio  $R_\pi$  in (6), whereas COMPASS has to consider the interference of the Coulomb and strong amplitude. The phase determined with the simple considerations in Ref. [28] for the Serpukhov experiment [5] is close to  $\pi/2$  meaning that the subtraction of a nuclear background assumed to be incoherent is justified. In the COMPASS analysis a Gaussian profile function is used for the diffractive background and the relative phase is determined by a fit to the cross sections. The contributions to the fit are not shown in Fig. 3(c) of Ref. [6]. With a more realistic "absorbing disc" for the profile function [29] all bumps in Fig. 3(c) are well reproduced and again a phase is close to  $\pi/2$  is indicated [30]. Without a real fit to the data it is impossible to estimate the effect of the model dependence of the diffractive background, but that it will have an influence is clear from Ref. [27]. Considering this situation the discussion of the central subtraction of the diffractive background is insufficient to get a feeling for model dependence of the analysis.

Comparison of data with different targets provide the possibility to check the  $Z^2$  dependence for the Primakoff cross section and to estimate a possible contribution of the nuclear background. Such an investigation was performed by the Serpukhov collaboration and they have obtained  $Z^2$  dependence with good enough accuracy. The COMPASS collaboration really have gotten their main result using only  $Ni$  target but they wrote that they also considered other targets on small statistic and obtained approximate  $\sim Z^2$  dependence.

It should be noted that in order to get an information about the pion polarizabilities, the authors considered the cross section of the process  $\gamma\pi^- \rightarrow \gamma\pi^-$  equal to the Born cross section and the interference of the Born amplitude with the pion polarizabilities only. The COMPASS collaboration analyzed this process up to the total energy  $W = 490$  MeV in the angular range  $0.15 > \cos\theta_{\gamma\pi}^{cm} > -1$ .

However, the contribution of the  $\sigma$ -meson to the cross section of the Compton scattering on the pion could be very substantial in this region of the energy and angles. Therefore, we consider this contribution.

#### IV. $\sigma$ -MESON CONTRIBUTION

The cross section of the elastic  $\gamma\pi$  scattering can be written as [20]:

$$\frac{d\sigma_{\gamma\pi \rightarrow \gamma\pi}}{d\Omega} = \frac{1}{256\pi^2} \frac{(s - \mu^2)^4}{s^3} \times [(1 - z)^2 |M_{++}|^2 + s^2(1 + z)^2 |M_{+-}|^2], \quad (8)$$

where  $z = \cos\theta_{\gamma\pi}^{cm}$ . The amplitudes  $M_{++}$  and  $M_{+-}$  have no kinematical singularities and zeros [31].

The dispersion relation (DR) for the amplitude  $M_{++}$  at fixed  $t$  with one subtraction was obtained in [20]:

$$\begin{aligned} ReM_{++}(s, t) &= Re\overline{M}_{++}(s = \mu^2, t) + B_{++} \\ &+ \frac{(s - \mu^2)}{\pi} P \int_{4\mu^2}^{\infty} ds' ImM_{++}(s', t) \\ &\times \left[ \frac{1}{(s' - s)(s' - \mu^2)} - \frac{1}{(s' - u)(s' - \mu^2 + t)} \right], \end{aligned} \quad (9)$$

where  $B_{++}$  is the Born term equal to

$$B_{++} = \frac{2e^2\mu^2}{(s - \mu^2)(u - \mu^2)}. \quad (10)$$

Via the cross symmetry this DR is identical to a DR with two subtraction. The subtraction function  $Re\overline{M}_{++}(s = \mu^2, t)$  was determined with help of the DR at fixed  $s = \mu^2$  with one subtraction where the subtraction constant was expressed through the difference  $(\alpha_1 - \beta_1)_{\pi^\pm}$ :

$$\begin{aligned} Re\overline{M}_{++}(s = \mu^2, t) &= 2\pi\mu(\alpha_1 - \beta_1)_{\pi^\pm} \\ &+ \frac{t}{\pi} \left\{ P \int_{4\mu^2}^{\infty} \frac{ImM_{++}(t', s = \mu^2) dt'}{t'(t' - t)} \right. \\ &\left. - P \int_{4\mu^2}^{\infty} \frac{ImM_{++}(s', u = \mu^2) ds'}{(s' - \mu^2)(s' - \mu^2 + t)} \right\}. \end{aligned} \quad (11)$$

The DRs for the amplitude  $M_{+-}(s, t)$  have the same expressions (9) and (11) with substitutions:  $ImM_{++} \rightarrow ImM_{+-}$ ,  $B_{++} \rightarrow B_{+-} = B_{++}/\mu^2$  and  $2\pi\mu(\alpha_1 - \beta_1)_{\pi^\pm} \rightarrow (2\pi/\mu)(\alpha_1 + \beta_1)_{\pi^\pm}$ .

The amplitudes  $M_{++}$  and  $M_{+-}$  were calculated with help of these DRs taking into account contribution of the following mesons:  $\rho(770)$ ,  $b_1(1235)$ ,  $a_1(1260)$ , and  $a_2(1320)$  mesons in the  $s$ -channel and  $\sigma(600)$ ,  $f_0(980)$ ,  $f'_0(1370)$ ,  $f_2(1270)$  mesons in the  $t$ -channel. It has been shown that the contribution of all these mesons, with

exception of the  $\sigma$ -meson, is very small in the region of the energy and the angles of the COMPASS experiment.

According DRs (11) the contribution of the  $\sigma$ -meson can be determined as

$$ReM_{++}^\sigma = \frac{t}{\pi} \int_{4\mu^2}^{\infty} \frac{ImM_{++}^\sigma(t', s = \mu^2) dt'}{t'(t' - t)}. \quad (12)$$

The imaginary amplitude  $ImM_{++}^\sigma(t, s = \mu^2)$  has to be evaluated taking into account that the  $\sigma$ -meson is a pole on the second Riemann sheet. The relation between amplitudes on the first and the second sheets can be written [32] as

$$F_0^{II}(t + i\epsilon) = F_0^I(t + i\epsilon)(1 + 2i\rho T_0^{II}(t + i\epsilon)), \quad (13)$$

where

$$T_0^{II} = -\frac{g_{\sigma\pi\pi}^2}{t_\sigma - t}, \quad F_0^{II} = \sqrt{2} \frac{g_{\sigma\gamma\gamma} g_{\sigma\pi\pi}}{t_\sigma - t}, \quad (14)$$

$$t_\sigma = (M_\sigma - i\Gamma_{\sigma 0}/2)^2, \quad \rho = \frac{\sqrt{1 - 4\mu^2/t}}{16\pi}, \quad (15)$$

$$\Gamma_{\sigma 0} = \Gamma_\sigma \left( \frac{t - 4\mu^2}{M_\sigma^2 - 4\mu^2} \right)^{1/2}.$$

Using the relation (13) we have

$$ImM_{++}^\sigma(t, s = \mu^2) = \frac{1}{t} \sqrt{\frac{2}{3}} \frac{g_{\sigma\gamma\gamma} g_{\sigma\pi\pi} R}{D^2 + R^2}, \quad (16)$$

where

$$D = (M_\sigma^2 - t - \frac{1}{4}\Gamma_{\sigma 0}^2), \quad R = M_\sigma \Gamma_{\sigma 0} + 2\rho g_{\sigma\pi\pi}^2. \quad (17)$$

We can get influence of the  $\sigma$ -meson on the extracted value of  $(\alpha_1 - \beta_1)_{\pi^\pm}$  by equating the cross section without  $\sigma$ -meson contribution to the cross section when  $\sigma$ -meson is taking into account:

$$\begin{aligned} & d\sigma_{\gamma\pi \rightarrow \gamma\pi}(B, (\alpha_1 - \beta_1)_{\pi^\pm}^0)/d\Omega \\ &= \sigma_{\gamma\pi \rightarrow \gamma\pi}(B, M_{++}^\sigma(\alpha_1 - \beta_1)_{\pi^\pm})/d\Omega, \end{aligned} \quad (18)$$

where  $(\alpha_1 - \beta_1)_{\pi^\pm}^0$  is the value of  $(\alpha_1 - \beta_1)_{\pi^\pm}$  without of the  $\sigma$  contribution obtained in [6]) and  $B$  is the Born term.

For backward scattering ( $z = -1$ ), we have the following expression:

$$\begin{aligned} & (\alpha_1 - \beta_1)_{\pi^\pm} \\ &= \frac{1}{4\pi\mu} \left\{ -(B + ReM_{++}^\sigma) + \frac{B^2 + 4\pi\mu B(\alpha_1 - \beta_1)_{\pi^\pm}^0}{B + ReM_{++}^\sigma} \right\}. \end{aligned} \quad (19)$$

In the case of integration over the region  $-1 \leq z \leq 0.15$  we have

$$(\alpha_1 - \beta_1)_{\pi^\pm} = F_0/F_1, \quad (20)$$

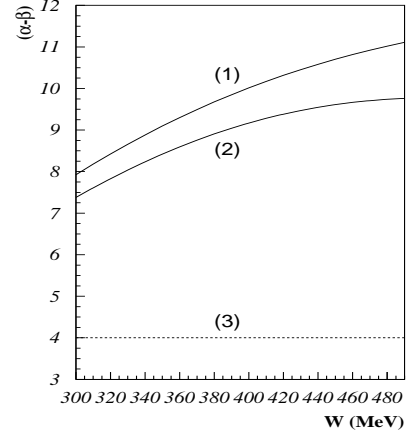


FIG. 1: Dependence of  $(\alpha_1 - \beta_1)_{\pi^\pm}$  on  $W$ . Lines (1) and (2) correspond to the calculation Eqs. (19) and (20), respectively. Line (3) is the COMPASS result [6].

where

$$\begin{aligned} F_0 &= \frac{1}{4\pi\mu} \left\{ \int_{-1}^{0.15} [-ReM_{++}^\sigma(ReM_{++}^\sigma + 2B) \right. \\ &\quad \left. + 4\pi\mu B(\alpha_1 - \beta_1)_{\pi^\pm}^0] (1 - z)^2 dz \right\}, \end{aligned} \quad (21)$$

$$F_1 = \left\{ \int_{-1}^{0.15} (B + ReM_{++}^\sigma)(1 - z)^2 dz \right\}. \quad (22)$$

In the calculation we used the parameters of the  $\sigma$ -meson from Ref. [32]:  $M_\sigma = 441\text{MeV}$ ,  $\Gamma_\sigma = 544\text{MeV}$ ,  $\Gamma_{\sigma\gamma\gamma} = 1.98\text{keV}$ ,  $g_{\sigma\pi\pi} = 3.31\text{GeV}$ ,  $g_{\sigma\gamma\gamma}^2 = 16\pi\Gamma_{\sigma\gamma\gamma}$ . The results of the calculations using Eq. (19) (line (1)) and Eq. (20) (line (2)) are shown in Fig. 1. Line (3) is the result of Ref. [6]. As a result we obtain  $(\alpha_1 - \beta_1)_{\pi^\pm} \sim 10$ . However the magnitude of  $(\alpha_1 - \beta_1)_{\pi^\pm}$  is very sensitive to parameters of the  $\sigma$ -meson and can reach a value of  $\sim 11$  for the parameters from [33].

So, the contribution of the  $\sigma$ -meson can essentially change the COMPASS result. It should be noted that the contribution of the  $\sigma$ -meson was not considered in Serpukhov as well. However, in this case, the contribution of the  $\sigma$ -meson for the Serpukhov kinematics is  $\Delta(\alpha - \beta)_\sigma \gtrsim 2.7$  within the experimental error of the Serpukhov result.

## V. TWO-PHOTON PRODUCTION OF PION PAIRS

The information about pion polarizabilities could be obtained also by studying the cross section of the reaction  $\gamma\gamma \rightarrow \pi^+\pi^-$ .

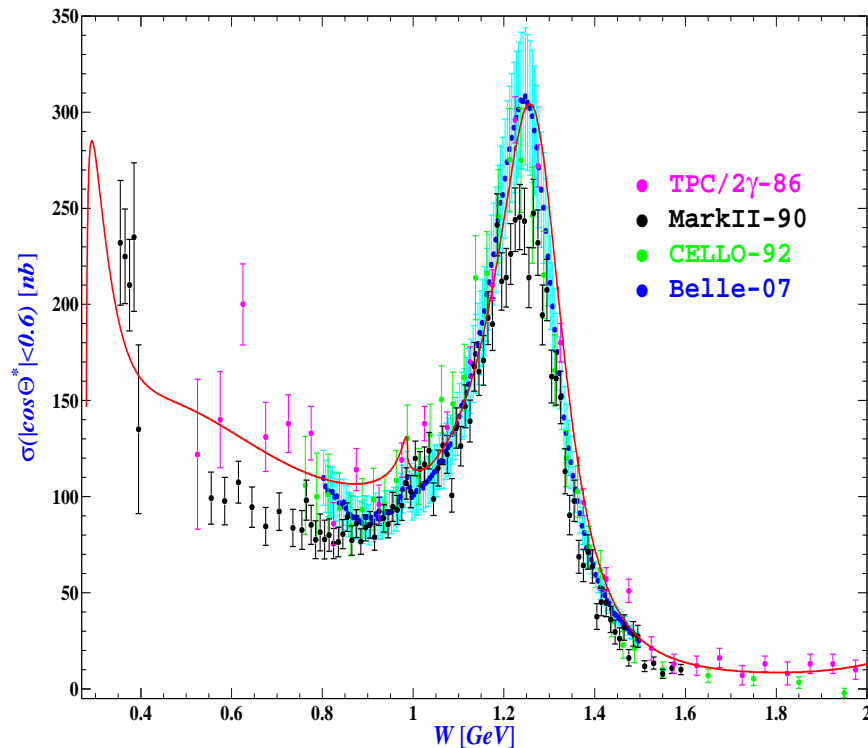


FIG. 2: The cross section of the process  $\gamma\gamma \rightarrow \pi^+\pi^-$  (with  $\cos\theta_{cm} < 0.6$ ). Experimental data of TPC/2 $\gamma$  [14], Mark II [10], and CELLO [15] Collaborations are shown with statistical uncertainties only. Statistical uncertainties for the most of the Belle Collaboration data [18] are smaller than corresponding blue circles. Vertical light-blue error bars are systematic uncertainties for these data.

Investigation of this process at low and middle energies was carried out in the frameworks of different theoretical models and, in particular, within dispersion relations.

Authors of most dispersion approaches used DRs for partial waves taking into account the contribution of  $S$  and  $D$  wave only. Moreover, they often used additional assumptions, for example, to determine subtraction constants. The dipole polarizabilities of charged pions were obtained in works [7, 11, 12, 19, 34, 35] from the analysis of the experimental data in the region of the low energy ( $W < 700$  MeV) mainly (where  $W$  is the total energy in  $\gamma\gamma$  c.m. system). The most of results for the charged pion polarizabilities obtained in these works are close to the ChPT prediction [22, 23]. On the other hand, the values of the experimental cross section of the process  $\gamma\gamma \rightarrow \pi^+\pi^-$  in this region are very ambiguous. Therefore, as it has been shown in Ref. [11, 13], even changes of these values by more than 100% are still compatible with the present error bars in the energy region considered. Therefore at present, more realistic values of the polarizabilities could be obtained analyzing the experimental data on  $\gamma\gamma \rightarrow \pi^+\pi^-$  in a wider energy region.

The processes  $\gamma\gamma \rightarrow \pi^0\pi^0$  and  $\gamma\gamma \rightarrow \pi^+\pi^-$  were analyzed in Ref. [2, 13, 20] using DRs with subtraction for the invariant amplitudes  $M_{++}$  and  $M_{+-}$  without an expansion over partial waves. The subtraction constants are uniquely determined in these works through the pion polarizabilities. The values of polarizabilities have been

found from the fit to the experimental data of the processes  $\gamma\gamma \rightarrow \pi^+\pi^-$  and  $\gamma\gamma \rightarrow \pi^0\pi^0$  up to 2500 MeV and 2250 MeV, correspondently. As a result the following values of  $(\alpha_1 - \beta_1)_{\pi^\pm} = 13.0^{+2.6}_{-1.9}$  and  $(\alpha_1 - \beta_1)_{\pi^0} = -1.6 \pm 2.2$  have been found in these works. In addition, for the first time there were obtained quadrupole polarizabilities for both charged and neutral pions.

The new result of the calculation of the total cross section of the process  $\gamma\gamma \rightarrow \pi^+\pi^-$  at  $\cos\theta_{\gamma\gamma}^{cm} < 0.6$  in the frame of the DRs [13] has been obtained with the  $\sigma$ -meson considered as a pole on the second Riemann sheet. The result, using Eq. (16) for  $ImM_{++}^\sigma(t, s = \mu^2)$  with the following parameters of the  $\sigma$ -meson:  $M_\sigma = 441$  MeV,  $\Gamma_\sigma = 544$  MeV,  $\Gamma_{\sigma\gamma\gamma} = 1$  keV,  $g_{\sigma\pi\pi} = 2.924$  GeV ( $\alpha_1 - \beta_1)_{\pi^\pm} = 10$ , is shown in Fig. 2.

However, the region  $W \lesssim 800$  MeV is most sensitive to the contribution of  $(\alpha_1 - \beta_1)_{\pi^\pm}$ . Therefore, in order to obtain real values of the polarizabilities of the charged pions, it is necessary to have new more accurate data for the process  $\gamma\gamma \rightarrow \pi^+\pi^-$  in the energy region  $W < 800$  MeV. It should be noted that even in the region of  $W \lesssim 500$  MeV the main contribution is given by the Born term, the dipole and quadrupole polarizabilities, and the  $\sigma$ -meson.

In conclusion, further experimental and theoretical investigations are needed to determine the true value of the pion polarizabilities. The authors thanks Th. Walcher and A.I. L'vov for useful discussions.

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